# THE STEADY STATE GROWTH RATE IN THE NEOCLASSICAL THEORY: A BRIEF SURVEY

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The so-called 'marginalist revolution' took place around the decade of the 70s of the 19th century. It produced a theory of the level and distribution of output based on the endowments of production factors, technology and consumer preferences. In such a theory, economic growth had to be conceived as the result of the increase in the endowments of factors. Early marginalist analyses of economic growth were developed by Alfred Marshall, Gustav Cassel and Knut Wicksell. We, however, start our survey from Robert Solow's (1956, 1957) formulation of the neoclassical growth model because it later became the basic point of reference for any discussion on neoclassical exogenous and endogenous growth.

In his 1987 Nobel Prize lecture, Robert Solow (1988) reconstructed the development of his research program on growth theory. He recalled how his work started by analyzing the Keynesian growth models by Roy Harrod (1939) and Evsey Domar (1946), and by attempting to solve their critical points. These economists had found that the economy could develop along a balanced growth path, provided that its productive capacity is utilized normally and the existing labour force is fully employed. First, these conditions require that investment (I) is equal to savings (S), the latter being a proportion of the level of output corresponding to normal capacity utilization. This gives us:

 $I=sY^*$ 

( $Y^*$  being the level of output corresponding to normal capacity utilization and s is the average propensity to save)

$$I/K = s/v = g_w$$

(Where K is the capital stock, v is the capital-income ratio and  $g_w$  is the so-called *warranted* growth rate)

The second requisite is that the warranted growth rate equals the growth rate of population (the natural growth rate) n. These two growth rates are not related in any definite way, and there is no guarantee that the autonomous investment decisions allow the economy to grow at the warranted growth rate. On the contrary, the growth path corresponding to  $g_w$  is, at least in Harrod's opinion, unstable (the so-called 'knife edge') so that the economy cannot grow along it.

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A solution to this kind of instability, which was perceived as highly unsatisfactory since it was not observable in the actual capitalist economies, has been provided by the first generation of neoclassical growth models.

## I EXOGENOUS GROWTH

The basic innovation of the Solow model has been the introduction of the neoclassical principle of substitution between the factors of production within growth theory. This principle enables investment to adjust to savings that corresponds to full capacity utilization, so that the economy grows at the warranted growth rate, and that the capital-income coefficient v varies until the warranted growth rate adjusts to the natural one, so that the economy employs the whole labor supply. This important advancement in growth theory is however responsible for one of its biggest pitfalls: the economy depends on parameters that are considered as given by theory, namely population growth and technical progress, and that it is unaffected by the economy's preference on accumulation (the saving rate).

This result follows from the variability of the capital-output ratio and the tendency towards full employment of all factors. To catch the underlying intuition, we can look at the growth rate of the economy as the result of the growth rates of capital (s/v) and labor (n). If the economy grows at a rate higher than n, then s/v needs to be higher than n. Capital will be growing faster than labor and the capital-labor ratio will be increasing. Given the decreasing marginal productivity of factors, the capital output ratio is also increasing. This means that s/v decreases until it adjusts to  $n^{-1}$ . Thus, sustained per-capita growth can be generated by assuming exogenous technical progress<sup>2</sup>. Technical progress is described as a factor of production on which theory has nothing to say. It is neither rewarded nor provided by any economic agent<sup>3</sup>. It is a public good because it is nonrivalrous and non-excludable, but there is no explanation for its production: it appears in the economy like 'manna from heaven'<sup>4</sup>. This seemingly unsatisfactory<sup>5</sup> characteristic of technical progress is strictly linked to the structure of neoclassical theory. From a logical point of view, the production function must be linearly homogeneous in labor and capital (the rival inputs), because of the so-called 'replication argument'. The introduction of a third factor (Romer (1990a), (1990b)) engenders increasing returns<sup>6</sup>. If this factor has to be the outcome of economic agents' decisions, it must be rewarded. But the neoclassical theory of distribution, under perfectly competitive conditions, cannot account for this reward. Under increasing returns to scale, the Euler theorem does not hold and the product is not sufficient to pay the factors of production according to their marginal productivity.

David Cass (1965) and Tjalling Koopmans (1965) provide a micro foundation for the neoclassical growth model by refining a seminal paper by Frank Ramsey (1928). Instead of assuming the propensity to save as a given fraction of national income, they derive the accumulation path of the economy by solving the inter-temporal maximization problem of a single (but representative) household. The conclusions of Solow's model on the exogeneity of the growth rate are unaffected.

In what follows, we analyze how the marginalist theory of growth made the technological progress and the steady state growth rate endogenous while maintaining its theory of distribution.

### II THE FIRST GENERATION OF ENDOGENOUS GROWTH MODELS

The basic device the new growth theories use to make the growth rate endogenous is through setting a lower bound to the decrease of the marginal productivity of capital, which equals, under perfectly competitive conditions, the profit rate. Indeed, in the Solow model the growth rate necessarily relies on exogenous technical progress because the marginal product of capital tends to zero and the incentive to accumulate necessarily converges towards zero too. There are basically three ways of stopping the decrease of the marginal product of capital: the introduction of a production externality, which offsets the decrease of the marginal productivity of private capital, the introduction of a 'human capital' factor, which assures that physical capital never becomes abundant, or by eliminating the factor through which the capital would become abundant, i.e. the labor supply.

#### a. The Externality

Kenneth Arrow (1962) suggested that technological progress can be ascribed to a process of learning by doing, and that the total stock of capital can be considered a measure of it. Basically, as accumulation goes on, the technological level of the economy increases. The problem of rewarding factors under increasing returns to scale is solved because the firms produce under constant returns to scale and they reward the factors of production according to their marginal productivity. At the same time, the accumulation of capital causes an externality on the technological level and therefore yields increasing returns to scale. The production function of the single firm i can be represented as:

$$Y_i = A(K)F(K_i, L_i)$$

(Where A represents the state of technology, K and L are capital and labor inputs with  $K=\Sigma i K i$ ).

Along those lines, the level of technology is taken as given through the individual choices on production, but it is endogenous because it depends on the level of accumulation and eventually on the propensity to save. The Arrow model, however, was developed in the case of a fixed capital-labor ratio and thus it implies that in the long run the output growth was limited by growth in population. The steady state output growth remained therefore independent of the savings behavior.

The same idea was later made popular by Paul Romer (1986). In an optimal growth framework, Romer introduces an externality into the production function due to the production of 'knowledge', i.e. the technological level. Knowledge (R) is produced by devoting resources to research. The externality arises because of the non-rivalrous nature of the knowledge input. Therefore the firm production function, assuming a Cobb-Douglas form, is:

$$Y_i = A L_i^{\ \beta} R_i^{1-\beta} R^{\gamma}$$
 where  $R = \sum_i R_i$ 

If  $\gamma = \beta$ , the aggregate production function becomes:

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$$Y = \sum_{i} Y_i = AL^{\beta} R$$

The production function exhibits constant return to scale in  $R^7$ . Since in these types of models *R* is commonly thought of as a sort of capital, we have derived a so called *AK* model. Its main characteristic is that the marginal productivity of capital is constant; the rate of growth (g) is given by the saved fraction of the net marginal product of capital, in our case:

$$g = s(AL^{\beta} - \delta)$$

(With  $AL^{\beta} = \frac{\partial Y}{\partial R}$  and where  $\delta$  is the depreciation rate)<sup>8</sup>

It must be made clear that the hypothesis  $\gamma = \beta$  is essential for the result of endogeneity of the steady state. The mere existence of the externality,  $\gamma > 0$ , i.e. increasing returns, is not sufficient to generate endogenous growth. From this point of view these kinds of models do not seem that robust.

An analogous result had already been put forward by Marvin Frankel (1961). He wanted to combine two different types of aggregate production functions: the Cobb-Douglas to account for the theory of distribution, and a linear production function  $a \ la$  Harrod (Y=aK) to re-establish the role of capital accumulation as the engine of growth.

The way he achieved this result is exactly the same way as Romer did. He introduced an externality of production (*E*), which is a function of the capital-labor ratio. The production function becomes  $Y = AK^{\alpha}L^{1-\alpha} E$ , if  $E = (K / L)^{1-\alpha}$  then Y = AK. It appears then that the novelty of the 'seminal' 1986 paper by Romer consisted only of focusing on the concept of knowledge instead of capital as the factor of production generating the externality. The lack of hype attributed to Frankel's contribution can probably be attributed to its reliance on the 'knife edge' assumption concerning the externality. Apparently this was not a sufficient reason to prevent Romer's paper from becoming a revolutionary one.

#### b. The Human Capital.

Human capital (H) constitutes the second line of research along which endogenous growth theories have developed. Robert Lucas (1988) developed and made popular the human capital growth model of Hirofumi Uzawa (1965). Lucas agrees with Romer that differences in productivity levels amongst countries results from differences in knowledge. But the source of knowledge is singled out more through the accumulation of human capital than in research.

The introduction of human capital to the model requires the definition of three different elements: the contribution of human capital (H) to production, the allocation of time between labour and the accumulation of H, and the link between the time devoted to accumulation of H and its rate of growth. The production function is:

$$Y = AK^{\beta} [uhL]^{1-\beta} h_a^{\gamma}.$$

(*u* is the share of time devoted to work and *h* the level of human capital of the individual worker. The last term  $h_a \gamma$  represents the externality due to the average level of human capital.)

The time, 1-u, devoted to the accumulation of H is related to its growth rate by the differential equation:

$$\dot{h} = \delta h^{\zeta} (1 - u)$$

Lucas stresses the necessity to rule out the case of  $\zeta < I$ , otherwise the model is unable to yield sustained growth. The case of  $\zeta > I$  yields explosive growth; therefore, he assumes  $\zeta = I$ : the production of *H* is linear in the level of human capital and is homogeneous of degree two in the two factors *H* and (*I-u*). In this case, the model generates sustained growth. It is worth noting that the existence of the positive externality is unnecessary in order to obtain sustained growth; the key assumption is the 'knife edge' condition  $\zeta = I^9$ . This assumption makes the *H* factor grow exponentially in a way depending on the agents' preferences between working and accumulating *H*: this is why the steady state growth rate of the economy becomes endogenous<sup>10</sup>.

Sustained growth with human capital had also been developed by Uzawa. Lucas (1988, pp. 19-20) acknowledges his contribution but stresses that Uzawa only described a path of optimal accumulation without obtaining it as the outcome equilibrium of the economy.

#### c. Constant Returns to Capital

This class of models is characterized by the same AK production function we have seen in Romer and Frankel. The rationale, however, is slightly different. It is assumed that all the factors of production are reproducible, i.e. they are all capital of some kind; then capital never becomes 'abundant' and its marginal productivity and the rate of profit never falls to zero. Sergio Rebelo (1991), for instance, assumes that output is produced by physical and human capital. Both kinds of capital are produced by means of a constant returns technology, which uses the same composite capital as an input.

Necessarily, the final good is also produced under constant returns and the assumption of a unique method of production makes the profit rate depend only on technology. If

$$Y = AK$$

Then

$$r = A$$

(Where r is the net rate of profit, which is equal to the marginal product of capital) and the endogenous rate of growth is

$$g = sA = sr$$
.

#### **III IMPERFECT COMPETITION**

Up to this point, the analyzed models of endogenous growth relied on either technical progress arising as a by-product of capital accumulation, or on the introduction of a second reproducible factor of production (human capital). The missing point is the analysis of technical progress as the outcome of intentional choices of profit (or utility) maximizing agents. The prolonged absence of this type of analysis, which contrasts with the evidence of firms' R&D expenses, can be explained by the difficulty of reconciling increasing returns and perfect competition. It follows from the 'replication argument' that the production function needs to have constant returns in labor and capital. If this is the case, and firms are price-takers, the reward of the two factors exhausts the whole product and there is no place for the reward of the research factor. It is therefore clear that the introduction of technical progress as the outcome of intentional choices implies the abandonment of perfect competition and the introduction of the idea that firms have market power. This is exactly the line of reasoning followed by Paul Romer in the development of his models from 1987 and 1990. These models are based on the idea that growth is sustained by the increased specialization of labor across increasing varieties of lines of production. The existence of increasing returns is the result of an increasing number of intermediate goods used to produce the final good. The production function is:

$$Y = L^{\alpha} \int_0^A x_i^{(1-\alpha)} di$$

(Where  $x_i$  is the quantity of the *i*th intermediate good and [0,A] is the interval on which the set of intermediate goods is measured)

The production set of the intermediate goods is non-convex. Indeed, fixed costs arising from acquiring or producing a new idea must be paid for by producing a new good. The intermediate sector is monopolistic since any firm is the exclusive beneficiary of an idea. Therefore, a firm chooses the value of x equating its marginal revenue, derived from taking the marginal product of the good as its demand price, to the marginal cost derived from a technology which uses only capital. If any unit of the intermediate good is produced by means of  $\eta$  units of capital (so that  $x_i \leftarrow \eta x_i$ ), we find a symmetric equilibrium with  $x_i=x$  and  $K=A\eta x$ . Plugging the expression for K in the production function:

$$Y = L^{\alpha} \int_{0}^{A} \left(\frac{K}{A\eta}\right)^{1-\alpha} di = L^{\alpha} K^{1-\alpha} \eta^{\alpha-1} A^{\alpha}$$

Thus, for given quantities of labor and capital, an increasing number of intermediate goods increases the quantity of the final good. Increasing returns are introduced through the differentiation of intermediate goods<sup>11</sup>. The dynamics of technological progress are explained by monopolistic rent seeking arising in the production of ideas so that, for the first time, it is the result of intentional and rewarded actions. The introduction of imperfect competition allows the existence of increasing returns to be handled.

Nevertheless, even in this case, assuming linearity in the production of the factor (A), which increases the technological level, is essential in order to obtain endogenous growth. Romer himself says: "Linearity in A is what makes unbounded growth possible,

and in this sense, unbounded growth is more like an assumption than a result of the model" (Romer (1990b) p. S84). The production function of ideas is represented as  $A = \delta L_a A$ , so that the increase of ideas in the economy is proportional to the stock of existing ideas. And this is also a necessary condition: "If A were replaced [...] by some concave function of A-[...] - human capital employed in research would shift out of research and into manufacturing as A becomes larger" (*ibid*.). Endogenous growth is therefore supported both by the existence of increasing returns and, in an essential way, by the existence of spillovers in research production. The rationale for this assumption is that knowledge is a non-rivalrous good, so that all the researchers can use it freely in their activity. But while the mere existence of this positive spillover seems plausible, the hypothesis that it can assure the linearity in the production of A is doubtful.

In a similar way, Gene Grossman and Elhanan Helpman (1991, chap.3) derive endogenous growth by means of a private sector engaged in R&D, which produces new ideas and increases the variety of the existing goods. The structure of the economy proposed here is analogous to that in Romer (1990), but the product differentiation belongs to the consumption sector instead of the intermediate one.

The contributions analyzed so far consider horizontal innovation, which consists of expanding product variety. The appearance of a new good introduces a new sector in the economy, which satisfies new functions and needs. The substitutability between the new product and those already existing is finite. Innovations, however, can be of a different nature. They often consist of quality improvements of already existing goods or of developing the production process of a certain good. This is vertical innovation and it is the basis of the Schumpeterian idea of 'creative destruction', where the destruction consists of rendering a series of goods obsolete.

Like in the horizontal case, the vertical innovation can be considered both for the final goods sector (Grossman and Helpman, 1991, chap. 4) and for the intermediate one (Aghion and Howitt, 1992 and 1998, chap. 2, 3). In the first case, there are N goods each of which can be qualitatively improved an infinite number of times. The producer owning the most updated version of a good becomes a monopolist. The profit flow deriving from being monopolistic constitutes the incentive to innovate. In the second case, there is a single final good produced under perfect competition and an intermediate sector in which the owner of the innovation of the last generation of the product is a monopolist. There are two basic ways in which vertical innovation differs from horizontal innovation.

First, risk is introduced into the production process of innovations. It is assumed that innovations arrive randomly according to a Poisson distribution of parameter $\lambda$ . The introduction of uncertainty makes the description of the innovation sector more plausible. Nevertheless, at the aggregate level, the growth rate of ideas is proportional to the arrival rate of the Poisson process  $\lambda n$  (*n* being the number of researchers), so that:

$$\frac{A}{A} = \lambda n \ln \gamma$$

(Where  $\gamma$  is a measure of the improvement in technology and the assumptions concerning the production of ideas are analogous to those of Romer)

Second, and more fundamentally, the vertical nature of innovation renders transitory any monopoly associated with an innovation. The discovery of a vertically integrated product determines the extinction of the previous monopolist's rents: this is the 'destructive' component of technological progress. The temporary nature of monopolistic rents is taken into account by a firm engaged in R&D so that there is an inverse relation between the investment in current research and its expected future amount. The 'knife edge' assumption on the production function of innovation, however, is unaffected.

## IV CONCLUDING REMARKS

Summing up, it seems that endogenous growth models are based on the hypothesis of constant returns in the reproducible factor - capital (physical or human) or knowledge. Such a structure implies a permanent influence of changes in the investment rates and in the share of labor devoted to R&D on per-capita growth rate and thus it opens large possibilities for economic policy. From a theoretical point of view these models are not robust in that they hinge on 'knife edge' restrictions on technology, which allow them to generate linearity in the production of the reproducible factor of production. This seems to be the reason why many of the results accomplished by the endogenous growth theories, even though they were already obtained during the 60's, were considered implausible<sup>12</sup>. The Solow model attempted to solve the problem of the Harrod-Domar instability and the new growth theories share the same feature. As Solow notes: "...modern literature is in part just a very complicated way of disguising the fact that it is going back to Domar, and, as with Domar, the rate of growth becomes endogenous" (1992, p.18).

Nevertheless, credit must be given to endogenous growth models because they have contributed to the focus on elements like the production of knowledge and human capital that had been underrated till then. The biggest accomplishment of those theories seems to be a technical one. By introducing general equilibrium models of imperfect competition, they made it possible for the neoclassical theory to cope with both increasing returns and innovation as the outcome of profit-maximizing agents' decisions<sup>13</sup>. From a neoclassical point of view, making innovation and the marginal theory of distribution consistent is a huge result; unlike Joseph Schumpeter's *Theory of Economic Development* (1934), where it is no longer necessary to assume that, whenever innovation occurs, the marginal theory of distribution breaks down.

#### **END NOTES**

1 This actually occurs when the production function satisfies some technical conditions (Inada (1963)). Early growth theorists were very aware of the possibility of deriving sustained growth by violating one of these conditions. For example, Solow ((1956) p.77) singled out the possibility of sustained growth when the elasticity of substitution is sufficiently high (see also Pitchford (1960), Arrow et al (1961) and Ferguson (1965)). The intuition is that the elasticity of substitution is a measure of the efficiency of the factors of production; then, if it is high enough, the incentive to accumulate does not vanish even asymptotically. Not surprisingly renewed attention has been devoted to the elasticity of substitution by the new (endogenous) growth theories (see Barro and Sala-y-Martin (1995)).

2 It must be noted that in order to derive a steady state growth rate, technical progress needs to be exponentially 'labor-augmenting'. That is to say that the technical change enters the production function as if it were a mere increase in the labor factor.

3 Shell ((1966), (1973)) provided models where innovation is a good produced in the economy. However he derives an optimal steady state level of technology (A), which in turn is not capable of engendering sustained growth.

4 Vintage capital models (see for example Solow (1960)) do not solve this problem. They require capital accumulation for the existence of technical progress but they do not explain who produces it.

5 '.. exogenous theories of technical change are essentially confession of ignorance..' (Shell (1973) p.77).

6 If *R* is the set of rival inputs, *N* a non-rival input and F(R,N) the production function, we have:  $F(\mu R, N) = \mu F(R, N)$ , so that  $F(\mu R, \mu N) > \_F(R, N)$  for  $\mu > I$  (See Romer (1990a)).

7 For the sake of simplicity, I have slightly modified the production function proposed by Romer by eliminating the physical capital input. Such a simplification does not alter the very nature of the function. In particular, as Romer pointed out later ((1994) p.15), it must be noticed that assuming a similar production function is not legitimate. Indeed, according to the 'replication argument', the production function needs to be fashion constant returns to scale in the rival inputs, i.e. the sole labour. Adding a factor of production should render the production function with increasing returns to scale. This conclusion has been avoided because of the difficulty in handling a similar case with respect to the distribution side. The solution suggested by Romer ((1987), (1990b)) is the introduction of imperfect competition in research growth models.

8 The steady state growth rate is obtained only if population is stationary, otherwise growth becomes explosive. In this class of models in fact, the size of the economy affects per capita growth rates. This problem is usually referred to as 'scale effect'. The literature on the topic, both empirical and theoretical, is huge and it would take us too far from the scope of our paper (see as a basic reference Jones (1995a), (1995b)).

9 It must be stressed however that the model, once the agents are allowed to consume 'leisure', becomes unable to generate sustained growth (Solow (1992)).

10 It can be noticed that the human capital factor enters the production function by simply increasing the amount of the labour factor; it therefore has exactly the same role as the labor augmenting technical progress.

11 More precisely, given the free entry condition, it is possible to compute the value of A in terms of K. By plugging the value in the production function, we obtain increasing returns in labour and capital.

12 As Stiglitz observes: "We knew how to construct models that 'worked', but felt uneasy making these special assumptions. It was one thing to assume that saving rates were constant- that was a behavioral hypothesis that provided a not bad description of the economy over long periods of time, (...). But it was quite another thing to assume, for instance, that the effects of learning just offset the effects of diminishing returns due to land scarcity! That was a technological assumption, and although we may have agreed with Einstein that God had created a universe of great simplicity, it seemed going too far to assume that he had endowed technology with these special parameters, simply so that we could construct our steady state models" (in Cesaratto (1999) p.790).

13 As admitted by Aghion himself: "The main contribution of the new growth theory so far has been predominantly technical in nature. It is now possible to deal with increasing returns and imperfect competition in dynamic general equilibrium models which are simple as those developed in the recent industrial organization literature. This technological breakthrough has in turn made it possible to formalize a number of existing ideas concerning development" ((1994), p.7).

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