Prediction and Description in Marxian Value Theory

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This paper reflects on recent work in Marxian value theory, with an emphasis on contributions by Foley and Shaikh, as well as those following their work. Sen's (1978) taxonomy of "predictive" and "descriptive" aspects of Marx's theory is adopted to appraise and discuss these recent contributions. Many have come to doubt Marx's theory of value as a predictive theory given the outcomes of the transformation problem debates, although recent work by Shaikh provides a forceful counter to this. Even if many remain skeptical of Marx's theory of value as a predictive theory, there are fruitful interpretations of Marx's theory as a descriptive theory, where the emphasis is on describing fundamental processes of capitalist economies like the allocation of social labor, capitalist competition, and exploitation. Recent work in Marxian value theory by Foley, Shaikh, and others has facets that are both predictive and descriptive. This short reflection highlights these and points to paths for future research.

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I. Introduction

Duncan Foley and Anwar Shaikh have made fundamental contributions to Marxian value theory, and their work shapes much of the landscape of current debates in the field. These contributions include: the role of money in Marx's theory of value and the so-called 'New Interpretation' (NI) of Marx (Foley 1982, 1986, 2005); approaches to the infamous 'transformation problem' (Foley 1982, 1986, 2011*b*; Shaikh 1977, 1981, 1982, 1984, 1992, 2016); classical-Marxian competition and profit rate dynamics (Scharfenaker and Foley 2017; Shaikh 1978, 1979, 1980*b*, 2008, 2016); and applications of the labor theory of value for understanding the dynamics of capitalist economies (Basu and Foley 2013; Foley 2011*c*, 2013, 2016, 2018; Shaikh 1998, 2016; Shaikh and Tonak 1994), among other contributions to Marxian economics more broadly. This paper provides a brief reflection on select key contributions by Foley and Shaikh to Marxian value theory. Attention is also paid to contributions by others following the work of Foley and Shaikh, as well as those working in similar directions. Foley and Shaikh's many contributions in other areas of classical-Marxian economics, as well as other areas of economics in general, are beyond the current scope.

Marx's theory of value, or what is often referred to as the labor theory of value (LTV), has many critics, both in mainstream and heterodox circles. One source of criticism is the transformation problem and issues of consistency between the labor content of commodities—or *labor values*—and prices, see for instance Samuelson (1970, 1971, 1974) and Steedman (1977), among others. A primary problem for critics is that labor values and prices cannot be shown to be theoretically consistent, and as a result, labor values do not determine prices, or are at best unnecessary for analyzing prices. Marx's theory of value has also been charged as being overly, or purely, "metaphysical" and lacking in substance for economic analyses (Robinson 1964).

The contributions by Foley and Shaikh and others, discussed here provide counters to these criticisms. Some argue that properly conceptualized notions of labor values (and the LTV) do bear a close relationship with prices, and thus are relevant for understanding the dynamics of equilibrium and relative

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prices.[1] Others take a different approach and reconsider the role of labor values and the LTV—as well as other categories which can be grouped under the heading of "labor accounts"—in Marxian economics (Cogliano, Veneziani and Yoshihara 2022).[2] In light of the vast literature responding to critics of Marx, in one way or another, it is important to note that there is no singularly unique way to interpret Marx's original work, and there are multiple "logically consistent" ways to approach Marxian value theory (Mohun and Veneziani 2017). Thus, despite the criticisms, there remain different interpretations of Marx's theory of value that yield robust insights into capitalist economies.

The typical view, of both critics and proponents of Marx, is that the LTV is a *predictive* theory since labor values should predict, or determine, prices. The outcome of the transformation problem debates has led many to doubt the LTV as a predictive theory. However, Shaikh's (1984; 1998; 2016) work, along with others, provides a forceful response to this view. Relatedly, even if labor values do not predict prices there are other useful predictive aspects of the LTV, particularly establishing a relationship between exploitation and profits.[3] Even if the LTV has usefulness as a predictive theory, "a strictly predictive interpretation of Marxian value theory is unnecessarily reductive" and needlessly limits the scope of analysis in Marxian economics (Mohun and Veneziani 2017, p.1415). Marx's theory of value may have limitations as a predictive theory, but there is an arguably fruitful approach to viewing Marx's theory of value as a *descriptive* theory. Thus, despite the critics, there are valid predictive and descriptive aspects of the LTV, and it is certainly not just metaphysical (Mohun and Veneziani 2017).

Sen (1978, p.175) identifies "predictive", "descriptive", and "normative" interpretations of Marx's theory that are "non-metaphysical".[4] Predictive interpretations of the LTV tend to focus on Marx's theory as a "theory of prices", but there is additional "predictive value" of the LTV in analyzing class and exploitation (Sen 1978, pp.175, 182). While treating the LTV as a theory of prices can run up against the problems raised in the transformation debates, the descriptive interpretation of Marx's theory can be quite compelling.

In Sen's (1978, p.177) analysis, Marx's theory of value can be taken as describing "the *activity* of production [original emphasis]" and "participation in production as personal participation through labour". This interpretation draws on Dobb (1937, 1973) and his emphasis on the *human* focus of Marx's ideas, particularly the central role assigned to "human productive activity" (Dobb 1973, p.145). As Sen (1978, p.177) summarizes, "Dobb saw in the labour theory of value a description of production that is focused on human beings—giving a quantitative expression to human effort". This approach adopts a broader notion of the concept of labor and a wider scope for economic analysis than standard approaches focused on prices and efficiency (Dobb 1973, p.145). A primary benefit of the descriptive interpretation of Marx's theory of value is that it highlights the central role of human labor effort in describing fundamental processes of capitalist economies like the activity of production, the allocation of social labor, capitalist competition, and exploitation (Cogliano et al. 2018; Cogliano 2023*a*).

These topics are of central concern in Marxian economics and the LTV can serve as a robust foundation for analyzing them. This paper reflects on the aforementioned contributions by Foley, Shaikh, and others with a focus on the aspects of these that align with treating Marx's theory of value as a descriptive theory.[5] This is not to downplay the predictive aspects of these contributions, but to emphasize what standard critiques of Marx miss. The aim here is to highlight the explanatory power

[1] See, for instance, Chilcote (2004), Cockshott and Cottrell (1997), Fröhlich (2013), Işikara and Mokre (2022), Ochoa (1989), Shaikh (1984, 1998, 2016), and Tsoulfidis and Maniatis (2002).

[2] See, for instance, Basu and Foley (2013), Duménil (1980, 1984), Cogliano (2018, 2023*a*,*b*), Cogliano et al. (2018), Cogliano, Veneziani and Yoshihara (2022), Foley (1982, 2000, 2011*a*,*b*,*c*, 2013, 2016, 2018), Flaschel, Franke and Veneziani (2013), Flaschel, Frölich and Veneziani (2013), Mohun (1994, 1996, 2004, 2005, 2006, 2009, 2013, 2014), Moseley (1986, 1988, 1990, 1991, 1993, 1994, 1998, 2000), and Yoshihara (2021), among others.

[3] See Veneziani and Yoshihara (2015, 2017*a*,*b*) and Yoshihara and Veneziani (2018) for recent axiomatic studies of the relationship between exploitation and profits, which are related to concepts from Duménil (1980) and Foley (1982).

[4] The normative interpretations of the LTV identified by Sen (1978) are beyond the scope of this paper. See Mohun and Veneziani (2017) and Cogliano, Veneziani and Yoshihara (2022) for discussion of the normative relevance of the LTV and labor values in measuring exploitation and inequality. See also Cogliano, Veneziani and Yoshihara (2016, 2019, 2021) for recent examples along these lines.

[5] It is worth noting that there are other areas of work by Shaikh that fit with the notion of descriptive theory used here. For instance, Shaikh's (1974; 1980*a*) work on aggregate production functions reveals their shortcomings for understanding distribution, and demonstrates how aggregate production functions provide poor descriptions of the labor process and other fundamental processes of capitalism. However, these contributions are beyond the scope of this paper.

of Marx's theory of value as a descriptive theory, and to briefly chart some paths forward for research. This is not a thorough or detailed survey, nor is the aim to rehash all the points of debate in Marxian value theory. This paper is meant to provide 'food for thought' for future research, and uses Foley and Shaikh's contributions as a foundation for discussion.

The remainder of the paper proceeds as follows. Section II presents the main issues raised by the transformation problem to establish the reference point for debates around Marx's theory of value while section III discusses aspects of Marx's method that are often overlooked in the transformation problem debates, as well as some related contributions by Foley and Shaikh that respond to critics of the LTV. Sections IV-VI discuss particular areas where the descriptive interpretation of the LTV is especially useful: section IV discusses capitalist competition; section V discusses labor mobility and the allocation of social labor; and section VI discusses recent work advocating the useful role of labor accounts in analyzing capitalist economies. Section VII concludes.

II. Labor Values, Prices, and Skepticism

The main issues raised by the transformation problem can be easily seen through the standard "dual-system" or "dualist" approach to labor values and prices (Foley 2000; Mohun and Veneziani 2017). The dual-system approach is often attributed to Morishima (1973) and Steedman (1977), among others, and it has become a reference point for much of the recent discourse in Marxian value theory. Both positive and negative perspectives on Marx's theory of value are often influenced, in some way, by the dual-system framework. The brief exposition here helps to clearly frame the sources of skepticism toward Marxian value theory, as well as the responses to the transformation debates that argue for the empirical and theoretical relevance of the LTV.[6] The presentation here is also helpful for defining concepts used throughout the remainder of this paper, and it frames the discussion of prediction and description in recent contributions in Marxian value theory.

Assume a linear circulating capital economy composed only of n productive industries, where each industry produces a single commodity and operates a single activity in production (i.e. there is no joint production and no multiple activities). Let each industry be denoted by $j = 1, \ldots, n$ and let prices be denoted by the $1 \times n$ row vector $\mathbf{p}$. Let $\mathbf{A}$ represent the $n \times n$ matrix of capital requirements for production (or non-labor inputs), with each element a_{ij} being the quantity of commodity i needed to produce a single unit of commodity j and column $\mathbf{A}_j$ denoting the capital requirements of industry j. Let $\mathbf{L}$ be the $1 \times n$ vector of direct labor requirements with element l_j. Let $\mathbf{b}$ be the $n \times 1$ subsistence bundle of workers, and let w and r, respectively, denote profit and wage rates. The gross output (or activities) vector for the economy is denoted by the $n \times 1$ vector $\mathbf{x}$, and the vector of net output is $\mathbf{y} = (\mathbf{I} - \mathbf{A})\,\mathbf{x}$, where $\mathbf{I}$ denotes the n-dimensional identity matrix. Lastly, let $\mathbf{v}$ denote the $1 \times n$ vector of labor values with element v_j. Under the dual-system, labor values are defined as $\mathbf{v} = \mathbf{vA} + \mathbf{L} = \mathbf{L}\,(\mathbf{I} - \mathbf{A})^{-1}$.

The transformation problem debates revolve around whether or not three relationships known as 'Marx's identities' can be shown to hold (Foley 2000). These identities deal with aggregate relationships across the n-commodity economy described above. They are: (i) total labor value is proportional to total price; (ii) total surplus value is proportional to total profit; and (iii) the profit rate calculated in labor values is equal to the profit rate calculated in price terms. These relationships cannot hold in the dual-system framework, except under restrictive scenarios.

First, whether one uses gross output or net output to determine total value and total price, although it is typical to use gross output, (i) cannot hold in general. Using gross output, total value is $\mathbf{vx}$, and using net output, it is $\mathbf{vy} = \mathbf{L}\,(\mathbf{I} - \mathbf{A})^{-1}\,(\mathbf{I} - \mathbf{A})\,\mathbf{x} = \mathbf{Lx}$. Similarly, total price is $\mathbf{px}$ or $\mathbf{py} = \mathbf{p}\,(\mathbf{I} - \mathbf{A})\,\mathbf{x}$ using gross or net output, respectively. For any gross output vector $\mathbf{x} > \mathbf{0}$, $\mathbf{vx}$ and $\mathbf{px}$, or $\mathbf{vy}$ and $\mathbf{py}$, are proportional if $\mathbf{p} = \phi\mathbf{v}$, where ϕ is some positive scalar. If prices are equilibrium prices, or *prices of production*, whether workers are paid after production, $\mathbf{p} = (1 + r)\mathbf{pA} + w\mathbf{L}$, or prior to production, $\mathbf{p} = (1 + r)(\mathbf{pA} + w\mathbf{L})$, for any $r > 0$, $\mathbf{p} \neq \phi\mathbf{v}$ as long as $\mathbf{A}_j/l_j$ are different for some $j = 1, \ldots, n$. Stated differently, equilibrium prices are proportional to labor values *if and only if* the profit rate is zero or organic compositions of capital across industries are uniform. A zero profit rate and

[6] See Desai (1988), Foley (2000), and Mohun and Veneziani (2017) for surveys of the transformation problem debates and developments in Marxian value theory.

uniform organic compositions of capital are somewhat restrictive—and arguably unrealistic—scenarios, thus proportionality between prices and labor values does not hold in general. Thus, $\mathbf{vx}$ and $\mathbf{px}$ will not be proportional and neither will $\mathbf{vy}$ and $\mathbf{py}$ under the conditions outlined here.

Similarly, for (ii), the lack of proportionality between labor values and prices means that total surplus value and total profit will not be proportional. Formally, total surplus value is $\mathbf{v}\left(\mathbf{I}-\mathbf{A}-\mathbf{bL}\right)\mathbf{x}$ and total profit is $\mathbf{p}\left(\mathbf{I}-\mathbf{A}\right)\mathbf{x}-w\mathbf{Lx}$, and these are not proportional so long as $\mathbf{p}\neq\phi\mathbf{v}$.[7] The same applies for (iii) and equality of the 'value profit rate' and 'price profit rate'. The value profit rate is $\frac{\mathbf{v}(\mathbf{I}-\mathbf{A}-\mathbf{bl})\mathbf{x}}{\mathbf{v}(\mathbf{A}+\mathbf{bL})\mathbf{x}}$ and the price profit rate is $\frac{\mathbf{p}(\mathbf{I}-\mathbf{A})\mathbf{x}-w\mathbf{Lx}}{\mathbf{pAx}+w\mathbf{Lx}}$, and these cannot be equal if $\mathbf{p}\neq\phi\mathbf{v}$.[8]

There are limited cases where Marx's identities can hold under the dual-system even if $\mathbf{p}\neq\phi\mathbf{v}$. As Morishima (1974) shows, if the gross output vector (or aggregator) is selected from along the so-called 'von Neumann ray', i.e. "the long-run equilibrium balanced-growth output vector", (i)-(iii) can hold.[9] Assuming the wage is equal to subsistence, $w=\mathbf{pb}$, and that wages are paid prior to production, the outlays required for production can be written as the matrix $\mathbf{M}=\mathbf{A}+\mathbf{bL}$. Let λ denote the largest positive eigenvalue of $\mathbf{M}$, and let $\bar{\mathbf{x}}$ denote the eigenvector associated with λ. Letting the subscript t denote time periods for production, prices can be written as $\mathbf{p}_t=(1+\bar{r})\mathbf{p}_{t-1}\mathbf{M}$, where $(1+\bar{r})=1/\lambda$. Taking $\bar{r}$ as the equilibrium profit rate for long-period prices, if initial prices are equal—or at least very close—to labor values, $\mathbf{p}_0=\mathbf{v}$, then the price vector will eventually converge to one where Marx's identities hold, provided $\bar{\mathbf{x}}$ is the gross output vector. Letting $\bar{\mathbf{p}}=\lim_{t\to\infty}\mathbf{p}_t$ where $\mathbf{p}_0=\mathbf{v}$, total value will be equal to total price, $\mathbf{v}\bar{\mathbf{x}}=\bar{\mathbf{p}}\bar{\mathbf{x}}$. Similarly, total surplus value will be equal to total profit, $\mathbf{v}\left(\mathbf{I}-\mathbf{M}\right)\bar{\mathbf{x}}=\bar{\mathbf{p}}\left(\mathbf{I}-\mathbf{M}\right)\bar{\mathbf{x}}$, and the value and price profit rates will be equal, $\frac{\mathbf{v}(\mathbf{I}-\mathbf{M})\bar{\mathbf{x}}}{\mathbf{vM}\bar{\mathbf{x}}}=\frac{\bar{\mathbf{p}}(\mathbf{I}-\mathbf{M})\bar{\mathbf{x}}}{\bar{\mathbf{p}}\mathbf{M}\bar{\mathbf{x}}}$. Thus, under Morishima's (1974) assumptions, (i)-(iii) can hold.

Morishima's (1974) results depend on assumptions about wages, initial prices, and the gross output vector.[10] The assumptions necessary for these results to hold could be seen as restrictive and Marx's identities will still fail to hold in more general circumstances. For the specific vector $\bar{\mathbf{x}}$, lack of proportionality between prices and labor values does not pose an issue. However, for *any* gross output vector $\mathbf{x}>\mathbf{0}$, the lack of proportionality between prices and labor values poses problems for Marx's identities. The inconsistency between labor values and prices, and problems maintaining Marx's identities, are a primary source of doubts about the LTV, particularly as a predictive theory.

These doubts have also led some to disregard the concept of labor values and other Marxian concepts like surplus value, and in general they have fueled questions about the LTV as a relevant basis for the Marxian notion of exploitation—with some going as far as concluding "that the relationship between the labor theory of value and the concept of exploitation is one of mutual irrelevance" (Cohen 1979, p.338). The rejection of labor accounts by many in the mainstream, as well as within heterodox circles, however, misses the usefulness of these concepts as part of a descriptive theory of capitalist economies.

As Foley (2000) summarizes, the outcome of the transformation problem debates in the second-half of the 20th century led to different responses by those who maintained the relevance of Marxian value theory, or at least some related Marxian concepts. One set of responses generally accepts the outcomes of the transformation debates and shows that a theory of exploitation, as well as a capitalist class structure, can be built on alternatives bases like differential ownership of wealth (or productive assets), without the need for consistency between labor values and prices—see for instance Roemer (1982) and others in what is sometimes called the 'Analytical Marxist' tradition.[11] However, other responses counter the generally received view on the transformation debates and establish both the theoretical and empirical relevance of labor accounts and Marxian value theory. Many of these contributions also highlight the human aspects of Marx's theory of value and its power as a descriptive theory.

[7] The lack of proportionality between total surplus value and total profit when prices are not proportional to labor values holds even if the wage is equal to the price of the subsistence bundle, i.e. $w=\mathbf{pb}$.

[8] Marx's identities also do not hold under other definitions of prices as long as prices are not proportional to labor values. See Wright (2014, 2019) for an example of an approach that redefines labor values such that proportionality and Marx's identities can hold.

[9] Thanks are owed to an anonymous referee for pointing out the nuanced importance of Morishima's (1974) main results for the transformation problem debates.

[10] It is worth noting that unless prices begin at or very close to labor values they will not converge to equilibrium prices that allow Marx's identities to hold.

[11] See Veneziani (2012) for a survey of both analytical and "rational choice Marxism".

III. Circulation, Money, and the Human Element

The conclusions of the dual-system approach are mathematically true. However, some may argue that the dual-system runs the risk of casting Marx's ideas in a "flat conceptual space", which overlooks important aspects of Marx's theory of value (Shaikh 1982, p.68). The rejection of labor values and labor accounts leads to a heavy focus on the properties of prices, and inherently misses descriptive aspects of Marx's ideas. Inconsistencies between prices and labor values withstanding, the "descriptive aspect" of Marx's theory of value "remains the capturing of the human dimension of production and distribution" (Sen 1978, p.183). Prices alone "lack this feature of describing personal participation" and the human dimension of Marx's theory (Sen 1978, p.179).

As Shaikh (1981, 1982) points out, human aspects like the labor process and production process fall into the backdrop in the dual-system, and the important "qualitative" features of Marx's theory of value can become lost. Similarly, as Foley (2016) argues, an overemphasis on prices cannot allow one to make a clear connection between the expenditure of human labor effort and value creation. Approaches adhering too closely to the mathematical properties of systems of prices and labor values can also miss one of Marx's key aims: theorizing the central tendencies of capitalist economies. This is one of the descriptive strengths of Marx's theory of value.

A. Marx's Long-Period Method

The formation of central tendencies in capitalist economies and their relationship to Marx's theory of value can be brought into clear focus through Marx's 'long-period method' (LPM) (Foley 2011*b*; Garegnani 1970, 1976, 1984; Kurz and Salvadori 1995). The LPM is an approach to understanding the inner laws and tendencies of capitalist economies that is shared across the classical political economists—notably Smith (2000) and Ricardo (1951)—and Marx. The LPM makes the abstract assumptions of labor and capital mobility to set aside temporary frictions in markets and/or historical abnormalities that may exist in reality so that capitalism's pure, or ideal, form can be analyzed—"the form that corresponds" to its "concept" (Marx 1981, p.291). The abstract assumptions of the LPM include consideration of a sufficient length of time so that the "real inner laws of capitalist production" can be seen as the "outcome of a whole series of protracted oscillations, which require a good deal of time before they are consolidated and balanced out" (Marx 1981, pp.266, 291). The sufficient length of time—i.e. the 'long-period'—is important for abstracting from vestiges of earlier modes of production that may affect capitalist social relations and labor and capital mobility.

In capitalist economies composed of many industries, workers expend labor effort producing value and surplus value, and are mobile across industries. Capitalists make decisions of how to allocate capital (i.e. where to invest capital), and capital is similarly mobile across industries. Labor mobility across industries induces a tendency for rates of exploitation to equalize as workers migrate across industries in response to variations in the amount of effort they need to expend relative to their compensation (Marx 1981; Foley 2016, 2018; Cogliano 2023*a*). If labor is assumed to be homogeneous and the length of the working day is uniform, labor mobility manifests as equalization of the wage rate, w, which is typical in many treatments of Marxian value theory, as in section II.[12] The movement of workers across industries is viewed as an open-ended process and the equalized rate of exploitation emerges as a center of gravity for ongoing fluctuations in rates of exploitation. Capital mobility means that capital moves from one industry to another in search of higher profit rates (or returns on investment) and so profit rates tend to equalize across industries. Capital exits industries with low profit rates and enters those with higher profit rates. The constant movement of capital across industries causes profit rates to continuously fluctuate and an equilibrium profit rate emerges as a center of gravity for these fluctuations (Marx 1981, pp.297-298). The equalization of the profit rate captures the classical-Marxian competitive process and leads to the formation of prices of production.

Marx elevates the tendencies for rates of exploitation and profit rates to, independently, equalize to the level of "economic laws"(Marx 1981, pp.275, 291). However, the long-period equilibrium with uniform rates of exploitation and profit rates is not necessarily a realized, or attainable, state of the

[12]See Cogliano (2023*a*) for detailed discussion of Marx's (1981, pp.241-242, 275) ideas about labor mobility and equalization of rates of exploitation, including the degree to which these ideas appear in the dual-system framework.

world given the concrete realities of capitalist economies. In reality, economies are in a constant state of flux and could operate above or below this equilibrium for extended periods of time. Similarly, historical and institutional barriers to labor and capital mobility could exist at any given time and take a variety of forms. In light of possible disruptions to labor and capital mobility, the LPM provides an analytical reference point for understanding the dynamics of capitalist economies—i.e. it captures the "law-like form" of capitalist economies (Marx 1981, p.291).

The abstract approach of the LPM brings clear focus to the inner laws and tendencies of capitalist economies so they can be understood independent of superficial phenomena. Much of the explanatory power of the Marxian perspective in describing underlying processes of capitalism—at a theoretical level—stems from the LPM and its abstract assumptions. The LPM also helps to highlight human aspects of Marx's ideas and the central role of human activity in describing the behavior of capitalist economies.

B. Circulation and Realization: Circuits of Capital and Revenue

One process that is helpfully framed by the LPM is the circulation and realization of value. The rejection of a concept of labor values by skeptics misses the importance of the circulation and realization of value, as well as competition, in Marx's theory (Shaikh 1981, p.300). In Marx's (1981, Ch.9) famous tables in volume three of *Capital*—which motivate the transformation problem debates—he shows how surplus value produced through the expenditure of productive labor effort across industries is realized as profit through the processes of circulation and competition. Capitalist competition tendentially leads to equalization of profit rates, and this equalized profit rate means that surplus value is realized in proportion to capital advanced. However, as Marx shows, surplus value is produced in proportion to the living labor engaged in an industry. Thus, as emphasized by Shaikh (1981, 1982) and Foley (2013), there are can be important differences between value and surplus value created at the point of production and their realized form of prices and profit.[13] Discrepancies between labor values and prices capture transfers of value taking place between the production and realization of value.

Shaikh (1981, 1982, 1992) provides a detailed discussion of how value transfers across the circuits of capital and revenue can lead to realized profit being different from actual surplus value produced. Total value produced can be conserved, even when prices are not proportional to labor values, and the particulars of the circulation and realization of commodities can cause surplus value to be different from profit. Shaikh (1981, 1982) illustrates this through an example of an economy with three departments: means of production, worker consumption goods, and capitalist consumption goods. In the first two departments transfers of value due to price-value deviations (i.e. $\mathbf{p} \neq \phi\mathbf{v}$) remain contained, or are accounted for, within the circuit of capital. However, price-value deviations in the third department, for example if price is below (above) value, cause a decline (increase) in value within the circuit of capital and an increase (decrease) in the value in the circuit of revenue, i.e. value is transferred between the circuits. This transfer causes total surplus value to differ from total realized profit, and the extent of these transfers depends on how much surplus value capitalists consume.

In Shaikh's approach, this "transfer of value between the circuit of capital and the circuit of revenue, through the process of exchange... explains why price-value deviations can give rise to deviations between the sum of profits and the sum of surplus-values, *without violating the law of the conservation of value through exchange* [original emphasis]" (Shaikh 1981, p.285). An implication of Shaikh's example is that the proportionality between total surplus value and total profit need not necessarily hold.[14] Thus, the rejection of labor values and labor accounts are possibly misguided and overlooks important descriptive aspects of the LTV.

A key point in Shaikh's presentation is that money expressions of value may necessarily differ from their underlying labor values, and, relatedly, dual-system calculations of prices from a given technology and wage obscure important processes of circulation and realization, and the role of labor values in these. Stated slightly differently, Steedman's (1977) argument that labor values are redundant because

[13] See Cogliano (2018, 2023*a*) for recent discussions of the importance of tracking differences between surplus value production and its realization.

[14] It is possible to argue that capitalist consumption of surplus value should be classified as part of total profit. However, debates along these lines are beyond the scope of this paper.

prices and labor values can be calculated from the same given data overlooks human aspects of Marx's ideas, particularly the production of value through the labor process and how prices of production form through processes of competition and realization.[15] Marx's LPM helps to highlight these processes in its description of the formation of central tendencies, and it supports the relevance of the LTV as a descriptive theory.

C. *Money, Exploitation, and Marx's Identities*

Shaikh's analysis of the circuits of capital and revenue raises a crucial question: in light of the transformation problem debates and skepticism toward Marx's theory of value, what aspects of Marx's theory and Marx's identities are most relevant, particularly in maintaining the human and descriptive dimensions of the LTV? The NI provides some insight into this question.

As mentioned in section III.A, in the Marxian framework, broadly speaking, the expenditure of human labor effort is the source of value, and in capitalist economies value takes on form of—or is realized as—money. The NI emphasizes this connection between human labor effort and money value added (Duménil 1980; Foley 1982, 1986; Mohun 1994, 2004). Specifically, the NI treats the "labor theory of value as the claim that the money value of the whole mass of net production of commodities expresses the expenditure of the total social labor in a commodity-producing economy" (Foley 1982, p.37). The connection between labor effort and money value is also intuitive based on Marx's description of *labor-power* as the "capacity for labour" and the "expenditure of human brains, muscle, nerves, hands, etc.", and the measurement of labor-power in units of socially necessary labor time, which comprise the value of commodities (Marx 1976, p.129, 134, 270).

In the NI, the relationship between money value and labor effort is summarized in the *monetary expression of labor time* (MELT), which has the units money value added per unit of labor time. The MELT allows conversion between magnitudes measured in monetary units and those measured in labor time, or labor values, in the aggregate or average cases. Another characteristic of the NI is that it treats the conservation of aggregate value across circulation and exchange as an axiom, or basic principle of the LTV. Letting μ denote the MELT, under the conservation axiom: $\mathbf{py} = \mu\mathbf{vy} = \mu\mathbf{Lx}$. Thus, μ facilitates proportionality between total price and total value (in terms of net output), as well as total living labor engaged in the economy, $\mathbf{Lx}$. In the n-commodity example used here, μ can be defined as $\mu = \mathbf{py}/\mathbf{Lx}$.

The NI also defines the *value of labor-power* as $w^{\mu} = w/\mu$, whereas the value of labor power under the dual-system is the labor embodied in the subsistence bundle, $\mathbf{vb}$. Under the definitions of the NI, one can then also convert between aggregate profit (measured in money units), $\Pi = \mathbf{p}(\mathbf{I} - \mathbf{A})\mathbf{x} - w\mathbf{Lx}$, and aggregate surplus value (measured in labor time), $S = (\mathbf{p}(\mathbf{I} - \mathbf{A})\mathbf{x} - w\mathbf{Lx})/\mu = \Pi/\mu$.

As outlined here, the NI allows two of Marx's identities to hold: total price is proportional to total value, and total profit is proportional to total surplus value.[16] However, rather than these identities holding as the result of a theoretical apparatus of prices and labor values, the identities hold by definition whether or not prices are proportional to labor values. Thus, the NI has the advantage that it "is completely general, in that it is consistent with *any* theory of price formation (including, but not restricted to, theories of profit-rate equalizing prices of production) [original emphasis]" (Foley 2000, p.23). This is not uncontroversial in the transformation problem literature, and there are some important differences between the NI and dual-system.

One key difference is treating *net product*, $\mathbf{y}$, as the relevant measure of output (or aggregator) in the proportionality of total price and total value. Some have taken issue with this aspect of the NI since Marx's (1981, Ch.9) treatment of labor values and prices includes proportionality between gross output

[15] See Shaikh (2016, pp.212-213, fn.7) for an additional example that highlights the human and social aspects of Marx's ideas that can be overlooked by dual-system approaches. See also Shaikh (1977, 2016) for more on Shaikh's approach to price-value deviations.

[16] Equality of the value and price profit rates can hold under the NI if one uses μ to convert money constant capital to its labor time equivalent. However, this treatment of constant capital and the relevance of a value profit rate hold an unsettled status in the literature following the NI. In the case of constant capital, Foley (2000, 2018) raises questions about the meaning of the labor time equivalent of constant capital, and in the case of the value profit rate, Mohun (2004) raises doubts about the concept itself.

and total value, which is also typical in the dual-system.[17] However, the focus on net product in the NI, arguably, maintains a more clear connection between value production and the expenditure of labor effort than other approaches. This connection can be seen in the aggregate relationship $\mathbf{py} = \mu\mathbf{vy} = \mu\mathbf{Lx}$, where the total value produced, $\mathbf{py}$, is proportional to the total living labor effort expended in commodity production, $\mathbf{Lx}$. Under the dual-system such a relationship does not hold in general, $\mathbf{px} \neq \mathbf{vx} \neq \mathbf{Lx}$, and the connection between value creation and labor effort is not as clear.

Under the NI's definitions, total profit and total surplus value are also proportional, and more importantly for our current discussion, surplus value corresponds to unpaid labor effort (Foley 2016). Since $\mathbf{py} = \mu\mathbf{Lx}$ and $S = (\mathbf{py} - w\mathbf{Lx})/\mu$, surplus value can also be written as $S = (\mathbf{Lx} - w\mathbf{Lx})/\mu$. The NI's definitions of surplus value and the value of labor power define exploitation "in terms of the monetary relations of the capitalist system of production" rather than simply equating the existence of exploitation with "the existence of a surplus product over the necessary consumption of a society" as in the dual-system (Foley 1982, p.43).[18] The "analytical danger" of equating exploitation with a surplus product is that a surplus product is a technical necessity for a growing economy, and a surplus product alone does not describe "the capitalist form of exploitation through money and the market" (Foley 1982, p.43).

A related conclusion of the transformation problem debates and the dual-system approach is that exploitation has limited meaning beyond being a determinant of positive aggregate profit by ensuring the existence of a surplus product. The association of positive profits with the existence of exploitation is summarized in the Fundamental Marxian Theorem (FMT) (Morishima 1973). An added strength of the NI's definition of exploitation is that exploitation is not just a determinant of aggregate profit or relegated to the FMT.[19] The NI's understanding of "exploitation in terms of the monetary relations of the capitalist system of production" allows exploitation to more readily be seen as "a factual description of a socio-economic relationship" (Foley 1982, p.43; Dobb 1973, p.145). Exploitation in the NI also, arguably, better frames class struggle as "a struggle not to eliminate the surplus product, but to transform the social relations that make the social surplus take the form of surplus value" (Foley 1982, p.43).

The NI's emphasis on the relationship between money value and labor effort leads to multiple conclusions that highlight the descriptive strengths of Marx's theory of value. In particular, that value creation is clearly associated with the expenditure of human labor effort, and that stating exploitation in terms of money relations captures real social relationships of capitalism. These also highlight the human dimension of the LTV and the importance of understanding capitalism through "personal participation through labour" (Sen 1978, p.177). It may be argued that the NI shifts the emphases found in the dual-system, and the approach does have its critics. However, the NI preserves key relationships in Marx's original work that align well with the descriptive interpretation of the LTV and Marx's focus on understanding key processes of capitalism.

IV. Competition and Profit Rates

The theory of competition and the behavior of profit rates is an area where the descriptive interpretation of the LTV is helpful for highlighting fundamental processes of capitalist economies. Competition in the classical-Marxian framework describes processes of the allocation of capital, the formation of equilibrium prices, and open-ended gravitation or ongoing fluctuations—as discussed in section III.A. The classical-Marxian competitive process is emphasized by Shaikh (1978, 1979, 1980*b*, 2008, 2016) across multiple contributions, and these interventions bring clarity to the competitive process in capitalist economies. As Shaikh (2016) and others—i.e. Clifton (1977)—explain, neoclassical (or conventional) theories of competition cannot capture the *process* of competition in the same way the classical-Marxian theory can. Even theories of market structures differing from perfect competition, such as imperfect

[17] Details of the debates about use of net product versus gross product are beyond the current scope. See Foley (2000), Moseley (2000), and Fine, Lapavitsas and Saad-Filho (2004) for discussion along these lines.

[18] The rate of exploitation in the dual-system can be written as $\frac{\mathbf{v}(\mathbf{I}-\mathbf{A}-\mathbf{bL})\mathbf{x}}{\mathbf{vbLx}}$, which reduces to $\frac{1}{\mathbf{vb}}-1$ if $\mathbf{v} = \mathbf{L}(\mathbf{I}-\mathbf{A})^{-1}$. As long as $\mathbf{vb} < 1$, $\frac{1}{\mathbf{vb}} - 1 > 0$.

[19] See Veneziani and Yoshihara (2015, 2017*a*,*b*) and Yoshihara and Veneziani (2018) for detailed analyses of the relationship between exploitation and profits in different approaches to Marxian value theory.

competition, can fall short since they are simply attempts to "make standard theory more realistic" and do not focus on processes of competition (Shaikh 2016, p.329).

A key contribution by Shaikh (1978, 1979, 1980*b*, 2008, 2016) is the refinement of the classical-Marxian theory of competition as the theory of "real competition".[20] In the theory of real competition individual capitalists compete both within and across industries, as competition is a "war of all against all" (Marx 1976, p.477). One feature of the theory of real competition is that it recognizes that individual industries are characterized by a structure of profit rates and capitalists within industries compete through costs and choices of techniques (Botwinick 1993, p.131). The price of the good produced by an industry will be governed by the "average" capital (Marx 1981, p.273): "This production price, as we explained earlier, is determined not by the individual cost price of any one industrialist producing by himself, but rather by the price that the commodity costs on average under the average conditions for capital in that whole sphere of production" (Marx 1981, p.779).

These "average conditions" that determine the price for an industry also determine the entry point for new capitalists in the industry. When capitalists compete across industries they enter industries under the "generally reproducible conditions" of production (Shaikh 2008, p.167). Any industry might be characterized by capitalists with some kind of unique cost advantage that yields them a higher-than-average profit rate, but the advantage possessed by these capitalists may not be replicable. For instance, Marx (1981, pp.779-785) uses the example of a capitalist with access to a waterfall as a source of power while other capitalists in the same industry rely on steam-engines for power since they do not have access to—or property rights over—the waterfall. Thus, new entrants into this industry will not be able to use the waterfall for power, they will need to use steam-engines or whatever the prevailing "average social conditions of production" might be (Marx 1981, p.780). In the theory of real competition, capitalists operating at these average conditions are defined as "regulating capitals" (Shaikh 2008, 2016; Botwinick 1993; Tsoulfidis and Tsaliki 2005).

Regulating capitals reflect the average conditions of production within and across industries, and they determine the profit rate on new investment in production. Thus, competition within industries and capital movement across industries will cause "the turbulent equalization of actual rates of profit on new investments" or "regulating profit rates" (Shaikh 2008, p.168).[21] The theory of real competition and the notion of a regulating capital have empirical merit, see for instance Mokre and Rehm (2020) and Tsoulfidis and Tsaliki (2005), but for the current focus, they also fit in with the descriptive approach to Marx's theory. Specifically, the theory of how competition *actually* operates within and across industries and what is a *realistic* profit rate to consider provide a granular description of the competitive process that other theories lack.

Competition and turbulent equalization of profit rates are fundamental processes in capitalist economies, and the classical-Marxian approach has a great deal of explanatory power for understanding these processes. The explanatory power of the classical-Marxian approach is not limited to description. Recent empirical treatments by Scharfenaker and Semieniuk (2017), Scharfenaker and Foley (2017), and dos Santos and Yang (2020), for example, show that distributions profit rates for firms across industries are "sharply peaked". These contributions focus on measurements of profit rates that are more standard in the classical-Marxian literature—i.e. returns on total invested capital as opposed to returns on new investment, as in studies of regulating capitals—and they develop novel empirical methods using tools of statistical mechanics.

A theme in the results is that profit rates fit sharply-peaked distributions with an understandable degree of dispersion around the peak, which can be seen as a reflection of the competitive process. Thus, statistically speaking, there is solid evidence for profit rate equalization with the equilibrium serving as a center of gravity. The information-theoretic approach deployed by the aforementioned contributions also has descriptive elements that align well with the overall focus of this paper. These approaches are able to develop a robust theory of how capital mobility decisions are made, which are the decisions underlying the actual process of competition in capitalist economies, and provide a description of how central tendencies like profit rate equalization are attained. Standard theories of competition do not

[20] See also Botwinick (1993) and Tsoulfidis and Tsaliki (2005).

[21] Regulating profit rates are sometimes also referred to as "incremental profit rates" or "incremental rates of return" (Duménil and Lévy 2012; Mokre and Rehm 2020).

capture underlying processes of capital mobility and competition, and the typical critiques of Marx's theory of value miss its descriptive validity.

The competitive process also plays a role in the issues of labor values and prices shown in the context of the transformation problem debates in sections II and III. Recall that Marx's (1981, Chs. 9, 10) identities are derived from a series of tables in volume three of *Capital* where he shows how surplus value is conserved in the competitive process and the formation of a general rate of profit and prices of production. Surplus value is produced in proportion to the living labor engaged in an industry, and surplus value produced in relatively labor-intensive industries is 'transferred' to relatively capital-intensive industries via competition such that a uniform r holds and prices of production, $\mathbf{p}$, form. Thus, the competitive process *creates* the differences between labor values, $\mathbf{v}$, and prices that lie at the heart of the transformation problem.

Marx (1981) is careful to point out how the process of competition has a distorting influence:

> *In competition, therefore, everything appears upside down.* The finished configuration of economic relations, as these are visible on the surface, in their actual existence, and therefore also in the notions with which the bearers and agents of these relations seek to gain an understanding of them, is very different from the configuration of their inner core, which is essential but concealed, and the concept corresponding to it [original emphasis] (Marx 1981, p.311).

The process by which prices of production form and the profit rate equalizes have this distorting effect and seem to "contradict both the determination of value by labour-time and the nature of surplus-value as consisting of unpaid surplus labour" (Marx 1981, p.311). This is precisely the issue on which critics of Marx's theory of value get stuck. The seeming lack of connection between $\mathbf{p}$ and $\mathbf{v}$ is not a problem with Marx's theory, it is a feature consistent with his vision of the competitive process. In fact, at a theoretical level, labor values and prices must be different. This difference is necessary not only to account for competition and profit rate equalization, it is also necessary for Marx's theory to be able to explain the more general case of unproductive activities (or unproductive labor), where the entirety of conventionally-measured value added for some industries (or industry-level GDP) is surplus value produced by other industries (Cogliano 2018, 2023*a*,*b*). The case of unproductive activities is discussed further in section V.

Despite the inconsistency between $\mathbf{p}$ and $\mathbf{v}$ at a theoretical level, measures of labor values and prices *can* be close empirically. As Shaikh (1984, 1998, 2016), Ochoa (1989), Chilcote (2004), Tsoulfidis and Maniatis (2002), and Işikara and Mokre (2022) show, *direct prices*—which are prices proportional to estimates of labor values from input-output data—are strongly correlated with observed market prices. These results have a great deal of predictive value in that the relationship between direct and market prices implies that the underlying labor content of commodities bears relevance for prices. These results are also a forceful counter to those who might criticize labor values as being 'metaphysical', or lacking in substance for applied analyses. The possible tension between the theoretical inconsistency of labor values and prices and the empirical studies mentioned immediately above is beyond the scope of this paper.[22] The empirical support for labor values provides a vindication for Marx's ideas, but labor values are also important for other aspects of Marxian analyses, as is discussed in section VI.

V. Labor Mobility and Social Labor

The obfuscating role of competition poses interesting problems for understanding the sources of value and surplus value production. As discussed in section III.C, the NI maintains a clear connection between total living labor and aggregate value production. In the aggregate, a relationship between human labor effort and value creation seems straightforward. However, at disaggregated levels this becomes more complicated. As Marx (1981, p.311) notes—and as discussed in section IV—the competitive process seems to undermine labor effort as the source of value and surplus value, and the industry level surplus

[22]See Torres-González and Yang (2019) and Torres-González (2022) for recent studies of the statistical properties of input-output data for the United States, which speak to some of the underlying reasons why direct prices and market prices can be empirically close. These studies find some tendency toward uniformity of organic compositions of capital, $\mathbf{A}_j/l_j$ for $j = 1, \ldots, n$, does exist and this can serve as part of an explanation for for the closeness of labor values and prices.

value produced can be different from surplus value realized as profit. Surplus value is produced in proportion to the living labor engaged in an industry, but in the long-period equilibrium, is realized in proportion to the capital advanced such that a uniform r holds.[23] The competitive process thus poses a problem for directly observing surplus value production at the industry level, or at more disaggregated levels.

In capitalist economies the money form of prices and profits are what can be directly observed, or are "visible surface phenomena", and "surplus-value and the rate of surplus-value are, relative to this, the invisible essence to be investigated" (Marx 1981, p.134). The problem of analyzing surplus value production at the industry level, as well as a broader understanding of the sources of value creation, is more of an '*inverse transformation problem*' where one needs to move from observable prices and profits (measured in money) to the underlying labor accounts of labor values and surplus value (measured in units of labor time). The inverse transformation problem was first identified by Robinson (1950), with the term first being used by Morishima and Seton (1961)—although the broader notion of uncovering labor accounts from observable data was implied even earlier by Rubin (1990). More recently, Foley (2011*a*) and Cogliano (2018, 2023*a*,*b*) have discussed the inverse transformation in detail, and proposed methods for undertaking the procedure.

An approach to the inverse transformation follows from the LPM and the abstract assumption of labor mobility. The labor and capital mobility in the LPM, respectively, lead to the independent tendencies for equalization of rates of exploitation and profit rates (Foley 2011*b*; Cogliano 2023*a*). Equalization of rates of exploitation means that workers evaluate the effort they need to expend in any job and the wages they receive in exchange for their effort. As Foley (2016) shows, balancing the ratio of labor effort to income is consistent with an equalized rate of exploitation, since surplus value represents surplus labor effort. Furthermore, the connection between ratios of labor effort to wages and rates of exploitation is intuitive considering Marx's (1981, pp.241-242) source for his ideas about labor mobility in Smith's (2000, Ch. 10) description of the equalization of the "advantages and disadvantages of labor". The connections between Smith and Marx, as well as equalization of rates of exploitation and the connection to labor effort are discussed in detail by Cogliano (2023*a*).

Marx's equalized rate of exploitation—abbreviated as EQRE—in conjunction with the conservation of value of the NI allows for an inverse transformation to be performed to recover surplus value production at the industry level. The aggregate rate of exploitation can be observed and calculated as the rate of surplus value, $\varepsilon = \frac{\mathbf{p}(\mathbf{I}-\mathbf{A})\mathbf{x}-w\mathbf{L}\mathbf{x}}{w\mathbf{L}\mathbf{x}} = \frac{\Pi}{w\mathbf{L}\mathbf{x}}$. Under the definitions of the NI, the rate of exploitation is known from ε by converting money magnitudes to their labor time equivalents using μ. The rate of exploitation is $e = \frac{(\mathbf{p}(\mathbf{I}-\mathbf{A})\mathbf{x}-w\mathbf{L}\mathbf{x})/\mu}{w\mathbf{L}\mathbf{x}/\mu} = \frac{\Pi/\mu}{w\mathbf{L}\mathbf{x}/\mu} = \frac{S}{w\mathbf{L}\mathbf{x}/\mu}$, and $e \equiv \varepsilon$. Marx also assumes that e is uniform across productive industries, or $e_j \equiv e$, all j, as a result of labor mobility. Equivalency of e and ε is shown in Marx's original tables in volume three of *Capital*, and as Cogliano (2023*a*) details, can arguably be seen as an additional 'identity' that much of the existing literature misses. Similarly, the rationale for uniformity of e_j is largely unrecognized in existing literature, even though it can be viewed as a "hidden part of the *definition* of 'value,' [original emphasis]" (Morishima and Seton 1961, p.206).[24]

If $\mu = \mathbf{p}\mathbf{y}/\mathbf{L}\mathbf{x}$, the rate of exploitation can be written as $e = \frac{\mu}{w} - 1$, and solving for the wage rate, $w = \mu\left(\frac{1}{1+e}\right)$. Let $\mathbf{s}$ be the industry-level surplus value produced per unit of output. Under the NI, surplus value production at the industry level can be written as the difference between labor performed and the labor time equivalent of the industry wage bill: $\mathbf{s} = \mathbf{L} - \frac{w}{\mu}\mathbf{L}$. Since $w = \mu\left(\frac{1}{1+e}\right)$, $\mathbf{s} = \mathbf{L}\left(1 - \frac{1}{1+e}\right)$, and again, surplus value production is proportional to the living labor engaged in

[23]This is apparent in both the dual-system framework shown in section II, and in the NI. In the dual-system the surplus value produced across industries per unit of output is $\mathbf{v}\,(\mathbf{I} - \mathbf{A} - \mathbf{b}\mathbf{L})$, and with $\mathbf{v} = \mathbf{L}(\mathbf{I} - \mathbf{A})^{-1}$, this becomes $(1 - \mathbf{v}\mathbf{b})\mathbf{L}$, whereas the profit (or realized surplus value) is $\mathbf{p}(\mathbf{I} - \mathbf{A}) - w\mathbf{L}$.

[24]See Cogliano (2023*a*) for an in-depth discussion of both aspects of Marx's EQRE, including its intellectual roots in the work of Smith (2000), and how it applies in both the NI and dual-system. In the dual-system, $\varepsilon \neq e$ since $\mathbf{p} \neq \phi\mathbf{v}$; formally, $\frac{\mathbf{p}(\mathbf{I}-\mathbf{A})\mathbf{x}-w\mathbf{L}\mathbf{x}}{w\mathbf{L}\mathbf{x}} \neq \frac{\mathbf{v}(\mathbf{I}-\mathbf{A}-\mathbf{b}\mathbf{L})\mathbf{x}}{\mathbf{v}\mathbf{b}\mathbf{L}\mathbf{x}}$. However, uniformity of e_j across productive industries is built into standard definitions of labor values, with industry-level rates of exploitation across the n productive industries all being $\frac{1}{\mathbf{v}\mathbf{b}} - 1$. See Cogliano (2023*a*) for further elaboration.

an industry. This reflects a foundational precept of the NI: the abstract labor content of production is proportional to the wage bill, or its labor time equivalent.

Recognizing the theoretical importance of labor mobility and role of e in estimating surplus value production is especially important in the more general case of unproductive industries. Contributions by Basu and Foley (2013), Foley (2011*c*, 2013), Mohun (2006, 2009, 2013, 2014), Moseley (1988, 1990, 1991), Paitaridis and Tsoulfidis (2012), and Shaikh and Tonak (1994), for example, show how considering a distinction between productive and unproductive industries—as well as possibly applying a similar distinction within industries—changes how categories like surplus value need to be calculated. Basu and Foley (2013) and Foley (2011*c*, 2013) provide detailed explanations of how entire industries—particularly those like the financial services, insurance, and real estate industries, among others—might be considered unproductive and "value-realising" as opposed to "value-adding".

If entire industries are unproductive they do not produce surplus value or any value at all, and the whole of their conventionally-measured value added (or industry-level GDP) is surplus value produced elsewhere in the economy. As Foley (2013, p.259) explains, "because competitively-determined prices of commodities may deviate from proportionality to the commodities' productive labor content, surplus value is not necessarily realized in the productive enterprises in which it is generated". Unproductive industries still participate in the competitive process that tends to equalize profit rates, thus their activities will contribute to the obfuscating effects of competition and complicate a clear understanding of labor effort as the source of value.

The NI and Marx's EQRE are further helpful for analyzing surplus value production in this more general case of unproductive industries. Suppose an economy has m total industries numbered $k = 1, \ldots, m$, with $n < m$ productive industries denoted by $j = 1, \ldots, n$ and $m - n$ unproductive industries denoted by $h = n + 1, \ldots, m$. Calculating ε to find e and perform an inverse transformation procedure now requires treating the conventionally-measured value added of the $m - n$ unproductive industries as part of the total surplus value, $S = \Pi/\mu$, but once this is done, ε can be found as the ratio of total profit to the total wages in productive industries (Cogliano 2018, 2023*b*). This calculation of ε will still be equivalent to e and uniformity of e_j across productive industries can be assumed to find industry-level surplus value production. Under the NI, the total abstract labor content of an industry's production is proportional to the wage bill, thus the wage bill reflects the total abstract living labor engaged in an industry. With this in mind, and because surplus value production is proportional to the living labor engaged in an industry, industry-level surplus value production can be calculated as $e\frac{W_j}{\mu}$, where W_j denotes the wage bill of productive industry j.[25] Using the matrix and vector notation from earlier, industry-level surplus value production in this case could also be written as $ew\mathbf{L}\widehat{\mathbf{x}}$, where $\widehat{\mathbf{x}}$ is the $n \times n$ diagonal matrix of the vector $\mathbf{x}$.

Marx's EQRE is not the only assumption that can be made to perform an inverse transformation procedure. There are also open questions as to the appropriate time horizon over which to assume this equalization might take place. However, *some* assumption needs to be made about the distribution of rates of exploitation to estimate labor accounts like surplus value and labor values from available data gathered in terms of money prices (Duménil, Foley and Lévy 2009; Foley 2011*a*; Cogliano 2018, 2023*a*,*b*). Szumski (1989) and Rieu and Park (2020, 2022), for example, explore alternatives to Marx's EQRE in undertaking an inverse transformation.[26]

It is also worth noting that the inverse transformation procedure—as outlined here—as well as the conversions between money and labor time magnitudes in the NI do not require that prices be equilibrium prices or prices of production: they can be performed with any given prices. Thus, the NI and EQRE allow estimation of surplus value production in very general cases.

The NI and Marx's EQRE provide a way to uncover surplus value production and labor effort as the source of value in general. The approach outlined here builds on Foley's (1982; 1986; 2000; 2005; 2011*b*) contributions to understanding the role of money in Marx's analysis, and offers a new role for labor accounts in Marxian value theory. Because surplus value production at the industry level is proportional

[25] See Cogliano (2018, 2023*b*) for recent applications of this method for estimating surplus value production to the case of the United States.

[26] The assumption of Marx's EQRE is at least implicit in the empirical treatments of labor values discussed in section IV.

to the living labor engaged in an industry, estimating surplus value using an inverse transformation method speaks to the allocation of productive social labor in capitalist economies. This perspective also reframes Marx's theory of value as a theory of the allocation of social labor, as opposed to a theory of prices (Foley 2016, 2018; Cogliano 2023*a*). This fits with the overall descriptive approach put forward in section I, where Marx's ideas about labor mobility and equalization of rates of exploitation can be seen as providing a description of the processes of the allocation of social labor that other theories do not capture.

This perspective further helps to clarify a link between labor and capital mobility and the formation of long-period prices, and connects to ideas found in Rubin (1990), particularly those about the determination of value and social need—or demand.[27] Seeing the LTV as a theory of the allocation of social labor shows how social need and aggregate demand can be important in Marx's theory of value, despite the common view that aggregate demand plays no role in Marx's work.

Rubin (1990, Ch.17) provides an overview of how the "volume of demand" plays a role in determining the amount of labor that is socially necessary to produce various commodities: "The multiplication of the value per unit of product (which is determined by the technical conditions of production) times the number of units which will be sold at the given value, expresses the social need which is able to pay for the given product [emphasis removed]" (Rubin 1990, pp.195-196). The social need for any particular commodity factors into the overall amount of social labor dedicated to production of that commodity. Extending this to all of capitalist production—or the economy as a whole—important connections between framing Marx's theory of value as a theory of the allocation of social labor and aggregate demand become more apparent: overall demand plays an important role in regulating quantities of "labor actually expended in given [branches] of industry" (Rubin 1990, p.185).

Rubin's approach is consistent with what is outlined above and the descriptive interpretation of the LTV. Rubin (1990, p.223) sees Marx's theory of value as describing capitalist economies as systems of "distributed labor" and "distributed capitals" in "dynamic equilibrium", where a key aim is to "see under the visible process of distribution of *capital* the invisible process of the distribution of social *labor* [original emphases]". Being able to connect an understanding of the distribution of social labor to demand is crucial for developing robust accounts of the dynamics of capitalist economies, particularly the organization of production and economic crises. The descriptive interpretation of the LTV, as well as ideas from the NI and Rubin, provide a promising framework for such analyses.

VI. Labor Accounts and Accumulation

There are other recent contributions that—similar to those in section V—reconsider the role of labor accounts in Marxian value theory. Contributions by Flaschel, Franke and Veneziani (2013), Cogliano et al. (2018), and Cogliano, Veneziani and Yoshihara (2022), for example, argue that labor values can serve as indicators of industry-level labor productivity, which are more reliable than conventional measures. The overarching aim of these contributions is similar to that of Cogliano's (2023*a*) and Foley's (2016) perspective that understanding labor mobility and surplus value production speaks to the allocation of social labor: industry-level labor productivity reflects the allocation of social labor and value-creating capacity across capitalist economies. These contributions also align with a descriptive approach to the LTV.

The arguments made by Flaschel, Franke and Veneziani (2013), Cogliano et al. (2018), and Cogliano, Veneziani and Yoshihara (2022) are based on Flaschel's (1983) concept of "actual labor values", which are defined as the average total labor content of commodities. This notion of labor values is conceptually similar to the NI, close to Shaikh's (1984; 1998; 2016), and notably different from definitions of labor values used by others, for example, Morishima (1974), Morishima and Catephores (1978), and Steedman (1977).[28]

[27] Thanks are owed to an anonymous referee for highlighting the relationship between Rubin's approach and that outlined earlier in this section.

[28] Morishima (1974) and Morishima and Catephores (1978) define labor values as the minimum amount (or efficient amount) of labor needed to produce a commodity. Steedman (1977) defines labor values as standard Leontief employment multipliers. See Cogliano et al. (2018) for more detailed discussion of different concepts and definitions of labor values, and see Yoshihara and Veneziani (2023) for an axiomatic treatment of measurements of labor content.

As technical progress takes place, driven by competitive pressures to accumulate, the labor content of commodities necessarily changes over time. Technical changes that are labor-saving and increase profitability will tend to reduce the overall labor content—or labor values—of commodities. Thus, over time, it should be expected that labor values exhibit a downward trend. Flaschel, Franke and Veneziani (2013) refer to this as the "Law of Decreasing Labor Content" (LDLC), and show that it holds for estimates of labor values based on German input-output data, while Cogliano, Veneziani and Yoshihara (2022) confirm the LDLC holds using U.S. input-output tables.

The changes in labor values captured by the LDLC act as good indicators of changes in labor productivity, as Marx (1976, p.131) notes "the greater the productivity of labour, the less the labour-time required to produce an article". Suppose that for some industry j an initial production technique can be described by $(\mathbf{A}_j, l_j)$ and the labor value of a unit of output of industry j is $v_j = \mathbf{v}\mathbf{A}_j + l_j$. Also suppose that a new more profitable technique is denoted by $(\mathbf{A}'_j, l'_j)$ with corresponding labor value $v'_j = \mathbf{v}'\mathbf{A}'_j + l'_j$, such that $v'_j < v_j$. This decrease in the labor value of output means that a unit of labor time applied in industry j has become more productive with the change from $(\mathbf{A}_j, l_j)$ to $(\mathbf{A}'_j, l'_j)$. This Marxian notion of labor productivity can be represented by the inverse of labor values, i.e. $\rho_j = 1/v_j$ and $\rho'_j = 1/v'_j$ with $\rho'_j > \rho_j$.

Labor values, or $\boldsymbol{\rho} = 1/\mathbf{v}$, are more reliable indicators of industry-level labor productivity than labor productivity measures based on conventionally-measured value added. Following Marx's arguments about the distorting effects of competition summarized in section IV and discussed in section V, conventional value added at the industry level (or GDP at the industry level) will include realized surplus value in the form of profit, not the surplus value that an industry actually produces. Thus, standard measures of industry-level labor productivity—i.e. conventionally-measured value added per worker—will not *really* capture labor productivity due to the differences between surplus value production and its realization in the form of profit. Marxian and conventional industry-level labor productivity can be quite different, as Flaschel, Franke and Veneziani (2013) and Cogliano, Veneziani and Yoshihara (2022) respectively show for Germany and the U.S.

Estimates of labor productivity using labor values are inherently counterfactuals given the assumptions needed in working with available input-output data—as are the estimates made using the inverse transformation method outlined in section V. However, despite their counterfactual nature, these estimates of labor accounts provide an important description of the underlying dynamics of accumulation and the allocation of social labor. Treating labor accounts in this way also, arguably, maintains important human dimensions of Marx's ideas through the emphasis on labor and value production.

VII. Conclusions and the Road Ahead

The debates around the transformation problem in the 1970s sowed doubts about the usefulness of labor accounts and Marxian value theory in general, among both mainstream and heterodox economists. These doubts place a heavy burden on those working in the Marxian tradition, as they are compelled to argue the relevance of Marx's ideas on two fronts. This is unfortunate given the merit the LTV has as a descriptive theory. In particular, the usefulness of labor accounts in explaining deep-seated processes and underlying dynamics of capitalist economies, and the human dimensions of Marx's work maintained by the descriptive approach. The works of Duncan Foley and Anwar Shaikh on Marx's theory of value make fundamental contributions along these lines.

In light of the developments in Marxian value theory in recent decades, adhering to the standard criticisms of the LTV is, arguably, no longer a tenable position. Foley's and Shaikh's contributions have done a great deal to shed light on the relevance of the LTV and labor accounts for understanding competition and profit rate dynamics, the role of money in capitalist social relations, and human labor effort as the source of value, among other topics. These contributions also leave multiple open avenues for future research in Marxian value theory. Possible areas for future work include, but are not limited to, further studies of price-value deviations and detailed studies of competitive dynamics from a Marxian perspective, the relationship between money and labor accounts, methods for estimating labor accounts from monetary data, and treating the LTV as a theory of the allocation of social labor. The contributions discussed in this paper and the promising lines for future research are a solid foundation from which to, hopefully, cast off the stale critiques of Marxian value theory.

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